On the effects of firing costs on employment and welfare in a duopoly market with entry*

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1. Introduction

The aim of this paper is to investigate whether the conventional detrimental effect that employment legislation would have on employment and overall welfare holds true in a strategic competitive environment, where an incumbent chooses its optimal output given the potential entry of a second firm.¹ As largely discussed in the literature, forms of employment protection legislation (EPL) vary across countries. Nevertheless, all such systems imply that firms have to bear some costs associated to firing workers.

While there appears to be no clear-cut empirical relationship between (average) unemployment rate and the degree of EPL legislation, from a theoretical perspective it can be argued that firing costs may have ambiguous effects on employment (Cahuc and Postel-Vinay, 2002). On the one hand, they are in fact beneficial for already employed workers, who enjoy a safer income source. On the other, however, excessive firing costs may lower the propensity of a firm to employ, given the rigidities that result from a frictional labour market. Thus, there

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¹ First quantitative measures on EPL in different countries have been provided by Bertola (1990) and Grubb and Wells (1993). See also Layard and Nickell (1999) and OECD (1999, 2004).

have been various concerns about the possibility of increasing the cost of laying off workers, in the light of such adverse effect.

It is even more difficult to ascertain the effects of an increase in firing costs on employment in a competitive environment, in which decisions on entry and output depend on those undertaken by the potential competitor. While employment is likely to benefit from stronger competition, as industry output overall increases, there may be some causes of concern for the existing firms. To accommodate new entrants, they would in fact have to reduce their scale of operation, thus bearing additional costs depending on the level of firing ones. It is therefore of interest to understand the role such costs play in this context.

In this paper, we study a two-stage duopoly game, with an incumbent and an entrant. At the first stage, the incumbent sets its output level for the period, while at the second the entrant decides whether to enter the market or not. Entry decision leads to competition on quantity (a là Cournot) between firms, so that the incumbent can decide whether to revise its production choices or leave them unchanged with respect to those of the first period. However, output reduction is costly due to the presence of EPL, which translates into firing costs: these influence entry decision, since the incumbent may be unwilling to downsize its production leading to deleterious effects on potential entrant's profits, as well as both firms' decisions in both periods. We show that employment and welfare actually increase with firing costs provided that these lie within a certain interval, over which firms' decisions are no longer related to the level of the EPL.

The intuition underlying our result is pretty much straightforward and can be exemplified as follows. In a dynamic environment, firing costs act as a commitment for the incumbent firm, which binds itself to produce a given quantity of output as long as the loss in profits coming from a production downsizing does not offset the costs implied by that downsizing, which are in fact due to EPL. Through such a commitment output, and hence employment, is thus stabilized, and it can be further argued that the final level of employment can, under some conditions, increase with firing costs. Moreover, overall welfare is maximised for a strictly positive level of firing costs.

This paper thus contributes to the literature that originates from the seminal works by Lazear (1990) and Bentolila and Bertola (1990) which investigates the relationship between firing costs and employment or, more broadly, social welfare. Lazear (1990) shows that firing costs have no effect on employment in a perfectly competitive labour market with flexible wages. Instead, Bentolila and Bertola (1990) consider the effects of firing costs on employment in the presence of rigid wages and uncertainty on (labour) productivity shocks. They compare a flexible economy, with no firing costs, and a rigid one, and show that while average long-run employment is left unaffected by such labour market frictions, short-run employment is higher in the rigid system, provided that the realized productivity shocks are small. The opposite holds when the latter are high instead.

More recently, EPL has been proved to be a "second best" solution in economies with risk-averse workers, insurance and capital markets imperfections (e.g. Pissarides, 2001). However, none of the above-mentioned works considers the role of strategic interaction among firms in imperfectly competitive markets in determining the relation between firing costs and employment.

In Lommerud and Straume (2012) the role of employment protection in the form of firing costs is analysed and compared against a different labour market institution, i.e. "flexicurity", in a monopolistic product market in the presence of a labour union. In such a framework, strategic interaction between the firm's output decision in the product market and the union's wage decision in the labour market is crucial and the role of firing costs in affecting those decisions is relevant. Lommerud and Straume's (2012) main focus is nonetheless on the effect of alternative labour market institutions on trade unions' incentives to oppose or endorse labour-saving technology and on firms' incentives to invest in such technology. Increased flexicurity - interpreted as less employment protection and a higher reservation wage for workers - unambiguously increases firms' incentives for technology adoption. Moreover, a higher reservation wage generally makes unions more willing to accept technological change, while less employment protection has the opposite effect, as this increases the downside (job losses) of labour-saving technology. However, the possibility for new firms entering the market is not contemplated, so that its effects on employment in the presence of firing costs (which represent the objective of this paper) are not investigated.

To the best of our knowledge, the paper which is most closed to ours is Majumdar and Saha (1998). They examine the implications of job security in the context of entry and wage bargaining. Assuming that production requires industry-specific skills, which can only be acquired through training, and that jobs are secure by law, the entrant will actually enter the market if and only if it can attract workers by offering a wage higher than the unionised wage paid by the incumbent. In this context, the authors show that job security may lead to an unexpected outcome, in which duopolistic competition conveys a lower welfare than the one attained in monopoly.

However, in Majumdar and Saha (1998) job security is more strictly defined as the incumbent's absolute impossibility to lay off workers, which is equivalent to assume that firing costs are infinite. Instead, we admit that firing costs, or the degree of EPL, can vary. This allows us to analyse in detail how firms modify their employment decisions with respect to different levels of firing costs, hence clarifying which type of – possibly non-monotonic – relationship actually holds between firing costs and employment.

The remainder of the paper is organized as follows. In the second section, we formally present our model. We split the discussion between the second and the first period and provide the main results. In the third section, we sketch some considerations on the effect of EPL on total employment, thus presenting the main results of the work. We end up with some conclusions and possible future research developments, while more technical details are relegated in a final appendix.

2. Model

We consider a simple dynamic framework with two stages (production periods) and two firms: firm 1 as the incumbent and firm 2 as the entrant. Firms face the following linear demand curve:

$$\mathbf{p} = \mathbf{a} - \mathbf{Q} \tag{1}$$

where p denotes the price, $Q = q_i + q_j$ total quantity, with i, $j \in (1,2)$, $i \neq j$, and a > 0 a positive parameter. Firms operate under constant returns to scale technology, using labour as the only input. We can thus normalize firm i's output in terms of the labour units it employs. Therefore, we have $q_i = l_i$ where l_i is the firm i's level of employment. Analogously, firms' total production costs are given

by $C_i = wl_i = wq_i$, where w is the wage. We assume that w is uniform across firms and periods².

We study the following game. At stage 1 (t = 1), the incumbent decides its level of output for the first period. At stage 2 (t = 2), the entrant decides whether to enter the market. If it decides to enter the market, it bears a fixed cost given by F. Then, the two firms compete à la Cournot, i.e. on quantities. Firm 1 will thus in general revise its production plan with respect to those whereof the first stage. However, employment protection, as it has already been discussed, makes it costly for the incumbent to downsize employment. Following Lommerud and Straume (2012), we model the degree of employment protection through a parameter $\phi > 0$ representing unitary firing costs that the incumbent must pay for each worker it wishes to dismiss with respect to the production level at the first stage. We solve the game through backward induction. To simplify the notation, we define k₁ the quantity produced by firm 1 at the first stage and q₁, q₂ the quantity of the two firms at stage 2. Furthermore, without loss of generality, we define $\alpha \equiv a - w$.

Clearly, if firm 2 does not enter the market and firm 1 is the sole operator, it is trivial to check that firm 1 acts as a monopolist in both production periods. This yields to the following equilibrium values for outputs and profits (which are constant across periods):

$$k_1 = q_1 = \frac{\alpha}{2}; \ \pi_1 = \frac{\alpha^2}{4}.$$

Let us consider instead the case in which firm 2 enters the market. Then, at stage 2, firms' profits are given by:

$$\pi_1 = \begin{cases} (\alpha - q_1 - q_2)q_1 - \phi(k_1 - q_1) & \text{if } q_1 \le k_1 \\ (\alpha - q_1 - q_2)q_1 & \text{if } q_1 > k_1 \end{cases},$$
(2)

$$\pi_2 = (\alpha - q_1 - q_2)q_2 - F.$$
(3)

Firms choose output to maximize (2) and (3), respectively. Given the quantity produced by firm 1, the reaction function of firm 2 is given by:

 $^{^{\}scriptscriptstyle 2}$ This can be interpreted as if the firms face a perfectly inelastic supply of labor schedule.

$$q_2(q_1) = \frac{\alpha - q_1}{2}.$$
 (4)

The reaction function for firm 1 is obtained given the quantity produced by firm 1 in the first stage and given the quantity produced by firm 2. This yields to³:

$$q_{1}(q_{2}) = \begin{cases} \frac{\alpha - q_{2} + \phi}{2} & \text{if } q_{1} < k_{1} \\ k_{1} & \text{if } q_{1} = k_{1} \\ \frac{\alpha - q_{2}}{2} & \text{if } q_{1} > k_{1} \end{cases}$$
(5)

The reaction functions whereof (4) and (5) are shown in Figure 1. At the second stage, instead, the equilibrium depends on the firm 1's level of output at the first stage. Indeed, by solving the system of equations (4) and (5), we get the equilibrium outputs for both firms 1 and 2:

$$q_{1}^{*} = \begin{cases} \frac{\alpha+2\varphi}{3} & \text{if } k_{1} > \frac{\alpha+2\varphi}{3} \\ k_{1} & \text{if } \frac{\alpha}{3} \le k_{1} \le \frac{\alpha+2\varphi}{3} \\ \frac{\alpha}{3} & \text{if } k_{1} < \frac{\alpha}{3} \end{cases}$$
(6)

$$q_{2}^{*} = \begin{cases} \frac{\alpha - \varphi}{3} & \text{if } k_{1} > \frac{\alpha + 2\varphi}{3} \\ \frac{\alpha - k_{1}}{2} & \text{if } \frac{\alpha}{3} \le k_{1} \le \frac{\alpha + 2\varphi}{3} \\ \frac{\alpha}{3} & \text{if } k_{1} < \frac{\alpha}{3} \end{cases}$$
(7)

³ Notice that this is similar to Dixit (1980).

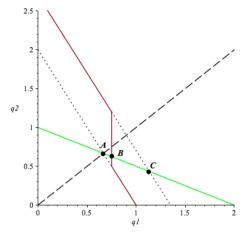


Figure 1. The reaction functions at the second stages. The three equilibria – A, B, C – represent the possible equilibrium outcome depending on the value of k_1 . In this case the realized equilibrium is $B = \left\{k_1, \frac{\alpha-k_1}{2}\right\}$. Value of parameters: $\alpha = 2, \varphi = 0.7, k_1 = 0.75$.

Hence, taking (1), (3), (6) and (7) into account, firm 2's profits are given by:

$$\pi_{2}^{*}(\mathbf{k}_{1}) = \begin{cases} \frac{(\alpha - \phi)^{2}}{9} & \text{if } \mathbf{k}_{1} > \frac{\alpha + 2\phi}{3} \\ \frac{(\alpha - \mathbf{k}_{1})^{2}}{4} & \text{if } \frac{\alpha}{3} \le \mathbf{k}_{1} \le \frac{\alpha + 2\phi}{3} \\ \frac{\alpha^{2}}{9} & \text{if } \mathbf{k}_{1} < \frac{\alpha}{3} \end{cases}$$
(9)

Figure 2 shows the equilibrium profits of firm 2 (without fixed costs) as a function of k_1 . Trivially, if $F > \frac{\alpha^2}{9}$, that is, the entry cost is larger than profits in a symmetric duopoly, firm 2 never enters the market. On the contrary, if $F < \frac{(\alpha - \phi)^2}{9}$, then firm 2 always enters the market, provided that $\phi < \alpha - 3\sqrt{F}$. Between these two values of F, there is a level of production at the first stage \hat{k} , such that if $k_1 \leq \hat{k} (k_1 > \hat{k})$, firms 2 enters (does not enter) the market. In what follows we assume that $F < \frac{(\alpha - \phi)^2}{9}^4$.

⁴ This assumption allows us to avoid the analysis of entry deterrence strategy. Although we think that this issue is very interesting, we focus on this simpler case to more clearly isolate the effect of firing costs in duopoly equilibrium.

Similarly, firm 1's profits at the second stage are:

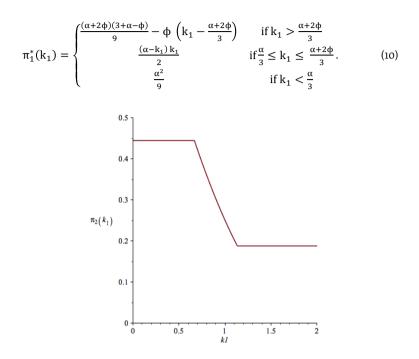


Figure 2. Gross profit at equilibrium of firm 2 as a function of the production level of firm 1 at the first stage (k_1). Values of parameters: $\alpha = 2$, $\varphi = 0$. 7.

At stage 1, the incumbent chooses output (employment) by maximizing the following profit function:

$$\Pi_1 = \nu_1(k_1) + \delta \,\pi_1^*(k_1) \tag{11}$$

where $v_1(k_1) \equiv (\alpha - k_1) k_1$ is the profit at the first stage (as a monopolist), and δ is the discount factor. In the first stage, firm 1 thus chooses k_1 to maximize (11) also taking into account the consequences that such choice yields with respect to stage 2. Total profit function is given by three segment of parabola that are determined by (10). This function is continuous and cannot take the maximum for $k_1 < \frac{\alpha}{3}$. We summarize these considerations as follows.

Remark 1

- For $k_1 < \frac{\alpha}{3}$, total profits are given by the function $(\alpha k_1) k_1 + \frac{\alpha^2}{9}$. The maximum is reached on $k_1 = \frac{\alpha}{2}$, and the profit is monotonically increasing until this value. However, since $\frac{\alpha}{2} > \frac{\alpha}{3}$, the only candidate is $k_1 = \frac{\alpha}{3}$.
- For $\frac{\alpha}{3} \le k_1 \le \frac{\alpha+2\phi}{3}$, total profits are given by $(\alpha k_1) k_1 + \frac{(\alpha k_1) k_1}{2}$. Again, this function takes its maximum value for $k_1 = \frac{\alpha}{2}$ and it is increasing for $\frac{\alpha}{3} \le k_1 < \frac{\alpha}{2}$. Thus $k_1 = \frac{\alpha}{3}$ cannot be a maximum, while candidates are $k_1 = \frac{\alpha}{2}$ (if $\frac{\alpha}{2} < \frac{\alpha+2\phi}{3}$) and $k_1 = \frac{\alpha+2\phi}{3}$ (if $\frac{\alpha}{2} \ge \frac{\alpha+2\phi}{3}$).
- For $k_1 > \frac{\alpha+2\phi}{3}$, total profits are given by $(\alpha k_1) k_1 + \frac{(\alpha+2\phi)(3+\alpha-\phi)}{9} \phi \left(k_1 \frac{\alpha+2\phi}{3}\right)$. This function takes its maximum value for $k_1 = \frac{\alpha-\delta\phi}{2}$. Hence, there are two candidates: $k_1 = \frac{\alpha+2\phi}{3}$ (if $\frac{\alpha-\delta\phi}{2} \le \frac{\alpha+2\phi}{3}$) and $k_1 = \frac{\alpha-\delta\phi}{2}$ (if $\frac{\alpha-\delta\phi}{2} > \frac{\alpha+2\phi}{3}$).

Accordingly, there are three candidates for the maximum: $k_1^A = \frac{\alpha}{2}$, $k_1^B = \frac{\alpha - \delta \phi}{2}$ and $k_1^C = \frac{\alpha + 2\phi}{3}$, where k_1^A and k_1^B are the maximum of the second and third segment of parabolas of the total profit function, and k_1^C is the interception between those two segments of the two parabolas. Figure 3 show the possible three cases that can emerge depending on the value of firing cost ϕ .

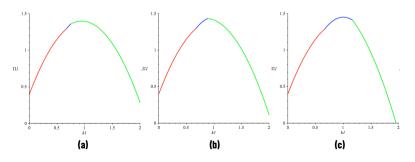


Figure 3. Firm 1's profit functions. Values of parameters: $\alpha = 2, \delta = 0.9$, (a): $\varphi = 0.125$; (b): $\varphi = 0.\overline{6}$; (c): $\varphi = 0.75$.

Proposition 1

If firing costs are low, that is $0 < \phi < \frac{\alpha}{4+3\delta}$, then optimal first and second period output is given by $k_1^* = \frac{\alpha-\delta\phi}{2}$, $q_1^* = \frac{\alpha+2\phi}{3}$, $q_2^* = \frac{\alpha-\phi}{3}$. For values of the firing costs lying in between the interval defined by $\frac{\alpha}{4+3\delta} \le \phi \le \frac{\alpha}{4}$, firm 1 does not modify its optimal output between the stages, which is given by $k_1^* = \frac{\alpha+2\phi}{3} = q_1^*$, while $q_2^* = \frac{\alpha-\phi}{3}$. For high firing costs, output is chosen regardless to such costs, yielding $\frac{\alpha}{4} < \phi$, $k_1^* = \frac{\alpha}{2} = q_1^*$, $q_2^* = \frac{\alpha}{4}$.

Proof:

See Appendix 1. ■

While the formal proof of Proposition 1 is provided in the final appendix, Figure 4 shows a graphical analysis of the results stated in the proposition.

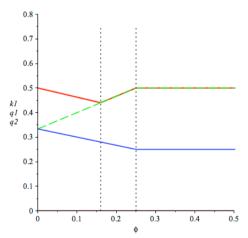


Figure 4. Equilibrium values of \mathbf{k}_1^* , \mathbf{q}_1^* , \mathbf{q}_2^* as a function of ϕ , red, green and blue respectively. The two vertical dashed thresholds are $\phi = \frac{\alpha}{4+3\delta}$ and $\phi = \frac{\alpha}{4}$. Values of parameters: $\alpha = 1, \delta = 0.75$.

Proposition 1 points out that the effects implied by the presence of firing costs are diametrically different depending on the period we consider. Indeed, stronger employment protection makes it more costly for the firm to operate with a large workforce in the first period, given the possibility of second-period downsizing in the light of enhanced competitive pressure. Thus, larger firing costs yield lower labour demand in the first period (Figure 3.a)⁵. However, this result only holds for relatively low levels of firing costs. Indeed, if ϕ increases above the threshold $\frac{\alpha}{4+3\delta}$, firm 1 can credibly sustain the same production level in both the periods (Figure 3.b). This quantity increases with ϕ until the Stackelberg equilibrium is reached (Figure 3.c).

In the second period, instead, the level of production of firm 1 increases with ϕ until $\frac{\alpha}{2}$. This increase is always sufficient to offset the decrease in production at the first stage. Indeed one can check that firm 1 wholly produces $k_1^* + q_1^* = \frac{\alpha - \delta \phi}{2} + \frac{\alpha + 2\phi}{3} = \frac{5\alpha + (4 - 3\delta)\phi}{6}$, which is always increasing in ϕ . Thus, total quantity produced by firm 1 is increasing for any $0 < \phi \leq \frac{\alpha}{4}$, and constant for $\phi > \frac{\alpha}{4}$. Moreover, firm 1's equilibrium profit can be easily obtained substituting k_1^* and q_1^* in the profit function. We get that profit are decreasing for $0 < \phi < \frac{\alpha}{9\delta + 16}$, as shown in figure 5.

Relative instead to firm 2, as shown in Figure 4, the quantity produced is decreasing in the firing costs for $0 < \phi \leq \frac{\alpha}{4}$ and constant for $\phi > \frac{\alpha}{4}$ and, given equation (9), also its profit follows the same path.

⁵ This parallels the result obtained by Lommerud and Straume (2012) in a monopoly model, where the possibility of second-period downsizing relates to the introduction of a new labor-saving technology. As stated by Lommerud and Straume (2012, p. 187), "This illustrates - in a very simple framework - the standard concern about the dynamic employment effects of employment protection legislation: if the cost of laying off workers is increased, this will make firms less willing to hire workers in the first place".

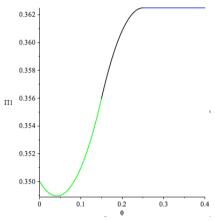


Figure 5. Firm 1's total profit as a function of firing costs ϕ . Values of parameters: $\alpha = 1, \delta = 0.9$

3. Employment and social welfare

In this section, we investigate the effects of firing costs on total employment. These are not clear-cut in our framework. As stated by Proposition 1, and given the assumption that $q_i = l_i$, unless $\phi > \frac{a}{4}$ (for which both firms' employment is not related to firing costs), employment dynamics are not monotonic with respect to the level of employment protection. The following proposition clarifies the effect of firing costs on total employment.

Proposition 2

The behaviour of second-period total employment with respect to firing costs parallels that of the incumbent's second-period output (employment): it is increasing for $\phi \leq \frac{\alpha}{4}$ and unrelated to firing costs for $\phi > \frac{\alpha}{4}$. Instead, overall employment (i.e. second-period total employment plus incumbent's first-period employment) is:

- decreasing in ϕ for low levels of firing costs, i.e. $0 \le \phi < \frac{\alpha}{4+3\delta}$, if and only if $\delta > \frac{2}{3}$.

- unrelated to ϕ for low level of firing costs if $\delta = \frac{2}{3}$ and for high levels of firing costs, i.e. $\phi \ge \frac{\alpha}{4+3\delta}$;
- increasing in ϕ for low level of firing costs if $\delta < \frac{2}{3}$, and for medium levels of firing costs, i.e. $\frac{\alpha}{4+3\delta} \le \phi < \frac{\alpha}{4}$.

Proof:

Proposition 1 shows that for $0 < \phi < \frac{\alpha}{4+3\delta}$, we have that $k_1^* = \frac{\alpha - \delta \phi}{2}$, $q_1^* = \frac{\alpha + 2\phi}{3}$, $q_2^* = \frac{\alpha - \phi}{3}$. Thus, we get that total employment $L = k_1^* + q_1^* + q_2^* = \frac{7\alpha + (2-3\delta)\phi}{6}$. It is easy to show that $\frac{\partial L}{\partial \phi} < 0 \Leftrightarrow \delta > \frac{2}{3}$. Following Proposition 1, for $\phi > \frac{\alpha}{4+3\delta}$, the results are trivial.

Corollary

From Proposition 2, it is easy to infer that the maximum level of total employment is obtained for $\phi > \frac{\alpha}{4}$, where $L = \frac{5}{4}\alpha$.

Relative to social welfare, we define it as the sum of consumer surplus and profits for the two stages, given a discount rate . That is:

$$SW = \frac{k_1^*}{2} + \delta \frac{(q_1^* + q_2^*)^2}{2} + \Pi_1 + \delta \pi_2^* .$$
(11)

Taking equilibrium values for k_1^* , q_1^* and q_2^* , as defined by Proposition 1, and the corresponding equilibrium profits, the following proposition applies for social welfare:

Proposition 3

The social welfare is:

- decreasing in the firing costs for $0 < \phi < \frac{\alpha}{4+3\delta'}$
- increasing in the firing costs for $\frac{\alpha}{4+3\delta} \le \phi < \frac{\alpha}{4}$

- constant for $\phi \ge \frac{\alpha}{4}$. Moreover, the maximum level of social welfare sustained as a Subgame Perfect Nash Equilibrium is obtained for $\phi = \frac{\alpha}{4}$.

Figure 6 provides a graphical proof of Proposition 3 and, in particular, that social welfare is maximised for a strictly positive level of firing costs (namely, $\phi = \frac{\alpha}{4}$)⁶.

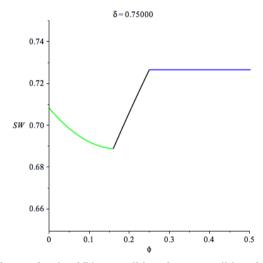


Figure 6. Social welfare as a function of firing costs. Values of parameters: Values of parameters: $\alpha=1,\delta=0.75$

4. Conclusion

⁶ This result can be checked by noting that the differential $SW|_{\phi=\alpha/4} - SW|_{\phi=0} = 7/144\alpha^2$ > 0.

In this paper, we aimed at providing a first step into the analysis on the role of firing costs in affecting employment and welfare in strategic competitive environments. In particular, we have analysed a duopoly model where an incumbent chooses its optimal output given the potential entry of a second firm. In such a context, the role of firing costs is not trivial since the incumbent may be unwilling to downsize its production and this can affect the potential entrant's decision. Indeed, we have shown that incumbent employment actually increases with firing costs provided that these are above a certain lower bound. Moreover, the total output and overall welfare in the market are strictly increasing in firing costs if the latter lie within a certain interval, over which firms' decisions are no longer related to the level of firing costs. This results can be explained by the fact that, in a dynamic environment, the incumbent exploits the presence of firing costs to credibly commit high level of production in the second stage, hence firing costs act as a commitment for the incumbent firm that contributes to stabilize output, and hence employment, which also leads to a result in which overall welfare is maximised for a strictly positive level of firing costs.

Since this paper has represented a first attempt to analyse the role of firing costs in a strategic environment, further research directed to extend our basic model and assess the robustness of our results can be carried out along possible different lines. For instance, analysing the effects of introducing strategic entry and endogenous wage determination under alternative unionization regimes, or comparing the effects of firing costs against different labour market institutional features (e.g. "flexicurity") deserve to be realized.

Appendix: proof of Proposition 1

From Remark 1 we get that total profits are maximized for $k_1^* = \frac{\alpha - \delta \phi}{2}$ when $\frac{\alpha - \delta \phi}{2} > \frac{\alpha + 2\phi}{3}$. Solving the inequality with respect to ϕ we get: $\frac{\alpha - \delta \phi}{2} > \frac{\alpha + 2\phi}{3} \Leftrightarrow \phi < \frac{\alpha}{4+3\delta}$. Hence, for $0 < \phi < \frac{\alpha}{4+3\delta}$, $k_1^* = \frac{\alpha - \delta \phi}{2}$ and, taking (6) and (7) into account, $q_1^* = \frac{\alpha + 2\phi}{3}$, $q_2^* = \frac{\alpha - \phi}{3}$. Instead, from Remark 1, we know that $k_1^* = \frac{\alpha+2\phi}{3}$ when $\frac{\alpha}{2} \ge \frac{\alpha+2\phi}{3}$ and $\frac{\alpha-\delta\phi}{2} \le \frac{\alpha+2\phi}{3}$. While, as shown above, the second inequality holds true for $\phi \ge \frac{\alpha}{4+3\delta}$, relative to the first inequality we get:

$$\frac{\alpha}{2} \ge \frac{\alpha + 2\phi}{3} \Leftrightarrow \phi \le \frac{\alpha}{4}$$

Hence, when $\frac{\alpha}{4+3\delta} \le \phi \le \frac{\alpha}{4}$, $k_1^* = \frac{\alpha+2\phi}{3}$ and, taking (6) into account, $k_1^* = q_1^* = \frac{\alpha+2\phi}{3}$ while, by substituting in (7), $q_2^* = \frac{\alpha-\phi}{3}$.

Finally, when firing costs are high, that is, $\phi > \frac{\alpha}{4}$, taking Remark 1 and above analysis into account, we get that $k_1^* = \frac{\alpha}{2}$. Moreover, since with $\phi > \frac{\alpha}{4}$, we have $\frac{\alpha}{3} < \frac{\alpha}{2} < \frac{\alpha+2\phi}{3}$, according to (6) we obtain $q_1^* = k_1^* = \frac{\alpha}{2}$ and, from (7), $q_2^* = \frac{\alpha}{4}$.

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